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Paper:

Suitable Aggregation Models Based on Risk Preferences for Supplier Selection and Order Allocation Problem

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In this paper, we propose (a) fuzzy multiple objective linear programming models for the Supplier Selection and Order Allocation (SSOA) problem under fuzzy demand and volume/quantity discount environments, and (b) an analysis of how to select the suitable aggregation operator based on the risk preferences of decision makers. The aggregation operators under consideration are additive, maximin, and augmented operators while the risk preferences are classified as risk-averse, risk-taking, and risk-neutral ones. The suitabilities of aggregation operators and risk preferences of decision makers are analyzed by a statistical technique, considering the average and the lowest satisfaction levels of the supplier selection criteria, based on numerical examples. Analysis results reveal that decision makers with different risk preferences will prefer only some aggregation operators and models. Moreover, a particular aggregation operator and model may generate a dominated solution for some situations. Thus, it should be applied with caution.

Keywords: fuzzy multiple objective linear programming, aggregation operators, risk preferences of decision makers, supplier selection and order allocation

1. Introduction

Selecting appropriate suppliers is one of the critical business decisions faced by purchasing managers, and it has a long term impact on a whole supply chain. For most firms, raw material costs account for up to 70% of product cost as observed in Ghodspour and O'Brien [1]. Thus, the supplier selection process is an important issue in strategic procurement to enhance the competitiveness of a firm. Effective selection of appropriate suppliers involves, not only scanning price lists, but also the requirements of organizations, which are increasingly important due to high competition in business markets. Typically, Dickson [2] indicated that major requirements are meeting customer demand, reducing cost, increasing product quality, and on time delivery performance. Hence, supplier selection is a Multi-Criteria Decision Making problem which includes both qualitative and quantitative data, and some of which may be conflicting. For conflicting criteria, decision makers need to compromise among criteria. To do so, decision criteria are transformed to objective functions or constraints. The relative importance (weight) of each criterion may be also applied to the model.

Essentially, to prevent a monopolistic supply base, as well as to meet all the requirements of firms, most firms have multiple suppliers which lead to the problem of how many units of each product should be allocated to each supplier. Thus, it becomes a Supplier Selection and Order Allocation (SSOA) problem.

Interestingly, to attract large order quantities, suppliers frequently offer trade discounts. Commonly, volume and quantity discounts are popular trade-discount strategies. The quantity discount policy aims to reduce unit cost, while the volume discount encourages firms to reduce the total purchasing cost. Both discounts are triggered at a certain purchasing level. For example, buyers purchase at $20 per unit (down from $25 per unit) when they purchase more than 100 units or receive a 10% discount when the total purchase cost of all products is greater than $1000. It is interesting to observe that the trade discount complicates the allocation of order quantities placed to suppliers. Thus, determining the different pricing conditions is a crucial task of decision makers to make the most beneficial buying decision.

Practically, firms try to place an order at the level of predicted demand to avoid excess inventory. However, when trade discounts are offered, firms usually purchase more than the predicted demand, to receive a lower price. Hence, to optimize the benefits, fuzzy demand is incorporated in models. Note that the satisfaction of demand criteria decreases whenever the order quantity deviates

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Table 1. A comparison of our study and other research works.

Author(s)	Fuzzy demand	Volume discount	Quantity discount	Multiple product
Xia and Wu [9]	No	Yes	No	Yes
Wang and Yang [10]	No	No	Yes	No
Amid et al. [11]	Yes	No	Yes	No
Lee et al. [12]	No	No	Yes	No
Zhang and Chen [13]	Yes	No	Yes	No
Suprasongsin et al. [14]	No	Yes	Yes	Yes
Hammami et al. [15]	No	No	Yes	No
Ayhan and Kilic [16]	No	No	Yes	Yes
Mazdeh et al. [17]	No	No	Yes	No
Cebi and Otay [18]	Yes	No	Yes	Yes
This model	**Yes**	**Yes**	**Yes**	**Yes**

from the predicted demand. Regarding the issue of uncertainty (fuzziness), fuzzy set theory (FST), developed by Zadeh [3], has been extensively used to deal with uncertain data, like in this case.

During the last decade, we have witnessed many decision techniques for handling multiple criteria decision making problem. Among several techniques suggested by Ho et al. [4], the linear weighting programming model proposed by Wind and Robinson [5], is widely applied to assess the performances of suppliers. The model is relatively easy to understand and implement. Later, with the use of pairwise comparisons, an analytical hierarchy process (AHP) allows a more accurate scoring method [6]. Generally, this technique decomposes the complex problem into multiple levels of a hierarchical structure. Similarly, Analytic Network Process (ANP), Goal Programming (GP), Neural Network (NN), etc., are also introduced to deal with the multiple criteria decision making problem.

In addition, a significant issue in the multiple criteria decision making problem is how to deal with different weights of criteria. Since, these weights are used in the model, weight aggregation operators are needed. Until recently, the most often used weight aggregation operator is weighted average operator or weighted additive operator. However, it has some drawbacks. It is not appropriate with interactive criteria. Thus, decision makers need to assume that all criteria are independent. This leads to some bias in making a decision. With the recognition of limitations, many scholars have developed advanced aggregation operators, such as the Choquet integral proposed by Schmeider [7] to deal with interactive criteria. The Ordered Weighted Averaging operator (OWA) introduced by Yager [8] is another popular aggregation operator used in the multiple criteria decision making problem. The OWA operator is actually the extension of the weighted average operator. The fundamental concept of OWA is that a weight is associated with the order of the score position, causing a non linear aggregation process. The Sugeno integral is also a well-known aggregation operator, which can be written in the form of a weighted max-min function. The weighted max-min function can be calculated as medians. In other words, it can be said that the Choquet integral is an extension of the weighted additive operator, while the Sugeno integral is an extension of the weighted max-min operator.

To simplify analyses, this paper focuses on the basic weighted aggregation operators, weighted additive operator, and weighted max-min operator, together with the weighted augmented operator. Note that the weighted augmented operator is the integration of weighted additive and weighted max-min operators. This paper also assumes that all criteria are independent.

Although several advanced techniques have been proposed to deal with the multiple criteria decision making problem, little attention has been shown as to which aggregation operator is suitable for a specific risk preference of a decision maker. Basically, the risk preference of decision makers can be classified into three types, namely, risk-taking, risk-averse, and risk-neutral. Another issue is that previous research works related to the SSOA problem have been conducted based on either volume or quantity discount, not both of them at the same time, as shown in **Table 1**.

Based on these motivations, this paper proposes realistic models with important practical constraints, especially volume and quantity discount constraints under fuzzy demand. Interestingly, three types of aggregation operators are applied to the models to determine which operator is suitable for risk-taking, risk-averse, and risk-neutral decision makers. The aggregation operators are (1) additive, (2) maximin, and (3) augmented operators. The models are developed from Amid et al. [11], Amid et al. [19], and Feyzan [20], accordingly. In addition, to test the sensitivity of the models, as well as the effect of aggregation operators, statistical analysis is conducted based on two performance indicators, namely, the average and the lowest satisfaction levels.

The rest of this paper is organized as follows. In Section 2, related terms are mentioned. Then, six developed models are presented in Section 3. In Section 4, statistical experiments are conducted to analyze the performances of the aggregation operators using MINITAB software. Results are discussed in Section 5, and some concluding remarks are presented in Section 6.

Table 2. Definition of attitudes toward risk of decision makers.

Indicator/Risk preference	Risk-taking	Risk-averse	Risk-neutral
Average satisfaction level	Highest	Any value	Not lowest
Lowest satisfaction level	Any value	Highest	Not lowest

2. Preliminaries

2.1. Aggregation Operators

To aggregate multiple criteria, many advanced aggregation operators have been proposed. However, in this paper, three basic types of operators are investigated with relative importance of criteria.

2.1.1. Additive Aggregation Operator

The weighted additive technique is probably the best known and widely used method for calculating the total score when multiple criteria are considered. In 2009, Amid et al. [11] applied this operator to their model where the objective function is:

$$\text{Max} \quad \sum_{i=1}^{I} w_i \lambda_i \quad \text{(1)}$$

where w_i is the relative importance of criteria i, and λ_i is the satisfaction level of criteria i. Note that to deal with multiple criteria, the dimensions of criteria are transformed to satisfaction levels, which are dimensionless.

2.1.2. Maximin Aggregation Operator

The goal of this operator is to maximize the minimum satisfaction level. In 2011, Amid et al. [19] introduced the model based on the maximin operator. In this model, s represents the smallest value of the criteria-satisfaction level.

$$\text{Max} \quad s \quad \text{(2)}$$

2.1.3. Augmented Aggregation Operator

In 2013, Feyzan [20] proposed this operator in order to keep the advantages of both the additive and maximin operators. The objective function is developed as follows.

$$\text{Max} \quad s + \sum_{i=1}^{I} w_i \lambda_i \quad \text{(3)}$$

2.2. Performance Indicators

Performance indicators are common tools for measuring the success of a target and are widely used in many fields. In this paper, a target is the explanation of the model's characteristics. The average and the lowest satisfaction levels of the supplier selection criteria are two characteristics for measuring the effectiveness of the models. In addition, a statistical technique has been employed in order to determine the significance of the findings.

2.3. Attitudes Toward Risks of Decision Makers

Real world decision making is usually made by people responsible for it. In order to understand how people make a decision, we need to know the nature of the people, so that we can select the most suitable decision making tool to fit with a particular type of people. In this paper, attitudes toward risk are used to classify the types of people. Generally, according to risk perception, people are classified into three types. Firstly, a risk-taking decision maker is one who enters into the risk as long as he/she possibly sees a positive high return. He/She might also be described as a decision maker who prefers the solution with relatively high value of average satisfaction levels of all criteria even though some criteria may have a very low or zero satisfaction level. The risk-taking decision maker feels that scarifying a criterion for the betterment of many other criteria is worth the risk. Secondly, a risk-averse decision maker, on the other hand, prefers to have as much certainty as possible, in order to reduce the discomfort level. He/She is very unhappy if the criterion has a very low or zero degree of satisfaction although many other criteria have a very high degree of satisfaction or high satisfaction level. Finally, a risk-neutral decision maker has a moderate opinion about risk. This type of risk preference decision maker feels that the average satisfaction levels of all criteria are important, and the lowest degree of satisfaction is important, too. Therefore, risk-neutral decision makers do not accept a solution with the lowest average satisfaction level or the one with the lowest value of the lowest satisfaction level. The attitudes towards risk of decision makers are summarized in **Table 2**.

2.4. Pareto-Optimality

A common problem addressed in multiple criteria decision making is how to compare the conflicting criteria in order to deliver a compromised Pareto optimal solution. Before going into details about what a Pareto optimal solution is, let us exemplify some fundamental concepts of the multiple criteria decision making problem.

Assume that Mr. Beta wants to buy a house. After considering many houses, he comes up with three choices, as shown in **Table 3**. In this case, his goals are to minimize the price, maximize his satisfaction in house design, and minimize the school distance of his children.

Based on this general example, we can clearly see that House B is more preferable than House C. Its price and distance are lower than House C. In addition, he also likes the design of the house more than House C. In this situation, we can say that House B dominates House C. Now let us consider House A and B. We can observe that both of them are incomparable since one is not better or worse

Table 3. A concept of multiple criteria comparison.

Alternatives/Criteria	Price ($\$\times 100000$)	Design (Score)	Distance (km)
House A	3	3.5	30
House B	3.5	5	20
House C	4	4	25

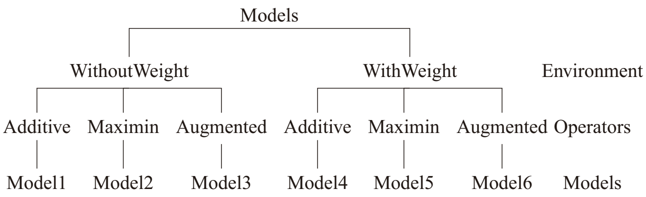

Fig. 1. A combined model diagram.

than the other in all criteria. This is a characteristic of a Pareto optimal solution or so-called, non-dominated solution.

In this paper, models are investigated, whether they generate the Pareto optimal solution or not. This is important because if a model's solution is dominated by other solutions, we may say that it is not a good model.

3. Model Development

There are six proposed models for the SSOA problem under fuzzy demand and volume/quantity discount constraints. Models under consideration are shown in **Fig. 1**.

3.1. Problem Description

In this study, decision makers must properly allocate the order quantities to each supplier so that maximum satisfaction is achieved. Four criteria are considered: (1) total cost, (2) quality of product, (3) delivery performance, and (4) preciseness of demand, where relative importances of criteria (weights) are given. The dominant effects among criteria are reduced by transforming them into satisfaction levels, in a range from 0.0 to 1.0. Demand of each product is allowed to be fuzzy. As multiple products are considered, the overall demand satisfaction level is the least satisfaction level of all products. The price-discount models were developed from Xia and Wu [9], Wang and Yang [10], and Suprasongsin et al. [14]. Note that the numerical data are given in **Tables 4–14**.

3.2. Mathematical Formulation

Let us assume that there are five products and five suppliers under consideration. Supplier k ($k = 1,\ldots,K$) offers either a volume discount or quantity discount when product j ($j = 1,\ldots,J$) is purchased at a discount level c ($c = 1,\ldots,C$). It is also assumed that supplier 3 offers a volume discount policy, while other suppliers offer a quantity discount policy.

Indices

i	index of criteria	$i = 1,\ldots,I$
j	index of products	$j = 1,\ldots,J$
k	index of suppliers	$k = 1,\ldots,K$
c	index of business volume breaks and price breaks' levels	$c = 1,\ldots,C$
m	index of fuzzy demand	$m = 1,\ldots,M$
n	index of demand (d) levels	$n = 1$ if $d \leq M$ $n = 2$ if $d \geq M$

Input parameters

dc_j predicted demand of product j (units)
h_{jk} capacity of product j from supplier k (units)
u_j maximum number of suppliers that can supply product j (suppliers)
l_j minimum number of suppliers that can supply product j (suppliers)
o_{jk} minimum order quantity of product j supplied from supplier k (units)
sr_{jk} 1 if supplier k supplies product j ; 0 otherwise (unitless)
r_{jk} minimum fraction of total demand of product j that has to be purchased from supplier k according to the agreement (percentage)
p_{cjk} price at discount level c of product j offered from supplier k (\$)
$z1_{jk}$ unit price of product j offered from supplier k (\$)
$z2_{jk}$ quality score of product j evaluated from supplier k (score)
$z3_{jk}$ delivery lateness of product j evaluated from supplier k (days)
e_{cjk} quantity break point of quantity discount at level c of product j from supplier k (units)
g_{ck} volume discount percentage from supplier k at discount level c (percentage)
b_{ck} dollar break point of volume discount at level c from supplier k (\$)
f_k 1 if supplier k offers quantity discount ; 0 otherwise (unitless)
w_i weight of criteria i (unitless)
σ weight of fuzzy demand (unitless)
mn_i minimum value of criteria i (\$, score, days)
md_i moderate value of criteria i (\$, score, days)
mx_i maximum value of criteria i (\$, score, days)
bo_{mj} boundary of demand level m of product j (units)

Decision variables

x_{cjkn} purchased quantity at discount level c of product j from supplier k at demand level n (units)
v_{cjk} purchased quantity at discount level c of product j from supplier k (units) at constant demand
π_{jk} 1 if supplier k supplies product j ; 0 otherwise (unitless)
t_{cjk} total purchasing cost j from supplier k at level c for quantity discount (\$)
a_{ck} total purchasing cost j from supplier k at level c for volume discount (\$)

Table 4. Weight sets (w_i, σ).

Factor/Weight	Weight set 1	Weight set 2
Cost	31%	38%
Quality	24%	28%
Delivery lateness	13%	11%
Demand	32%	23%

Table 5. Predicted demand (dc_j).

Product	Predicted demand
1	500
2	30
3	100
4	700
5	2500

Table 6. Narrow (N) and wide (W) demand range (bo_{mj}).

Level/Product	P1		P2		P3		P4		P5	
	N	W	N	W	N	W	N	W	N	W
Minimum variation	450	100	25	10	50	20	650	200	2300	1500
Predicted demand	500	500	30	30	100	100	700	700	2500	2500
Maximum variation	550	1000	32	80	160	500	720	1500	3000	5000

Table 7. Unit (List) price, quality score, and delivery lateness for incomplete trade-off (I) and complete trade-off (C); ($z1_{jk}$), ($z2_{jk}$), and ($z3_{jk}$).

Data	P/S	S1		S2		S3		S4		S5	
		I	C	I	C	I	C	I	C	I	C
Unit (List) Price	P1	50	50	40	40	55	55	50	50	45	45
	P2	0	0	200	200	0	0	230	230	0	0
	P3	70	70	75	75	72	***69***	0	0	0	0
	P4	0	0	0	0	8	8	10	10	5	5
	P5	0	0	0	0	0	0	20	20	20	20
Quality score	P1	3	3	5	8	6	6	2	2	4	4
	P2	0	0	6	6	0	0	7	7	0	0
	P3	5	5	7	7	6	***8***	0	0	0	0
	P4	0	0	0	0	8	8	10	10	5	5
	P5	0	0	0	0	0	0	8	8	9	9
Delivery lateness	P1	3	3	1	1	2	2	4	4	3	3
	P2	0	0	4	4	0	0	3	3	0	0
	P3	2	2	2	2	1	1	0	0	0	0
	P4	0	0	0	0	3	3	5	5	4	4
	P5	0	0	0	0	0	0	5	5	3	3

Table 8. Limited number of supplier (u_j, l_j).

No. of supplier	P1	P2	P3	P4	P5
Maximum	2	5	3	4	3
Minimum	1	1	1	1	1

Table 9. Break point of volume discount (b_{ck}) and volume discount percentage (g_{ck}).

Level	Supplier 3	
	b_{ck}	g_{ck}
1	0	0
2	10000	0.05
3	50000	0.1

Table 10. Available supplier for each product (sr_{jk}).

P/S	S1	S2	S3	S4	S5
1	1	1	1	1	1
2	0	1	0	1	0
3	1	1	1	0	0
4	0	0	1	1	1
5	0	0	0	1	1

Table 11. Price of each product for quantity discount levels (p_{cjk}).

Level/Supplier	S1			S2				S3					S5			
	P1	P3	P2,4,5	P1	P2	P3	P4-5	P1	P2	P3	P4	P5	P1	P2-3	P4	P5
Level 1	50	70	0	40	200	75	0	50	230	0	32	20	45	0	29	20
Level 2	45	68	0	39	180	74	0	48	220	0	30	18	43	0	28	17
Level 3	43	65	0	38	170	73	0	46	210	0	28	16	42	0	25	14

Table 12. Break point of quantity discount at level (e_{cjk}).

Level/	S1	S2		S4		S5
Supplier	P1-5	P1,3,4,5	P2	P1,3,4,5	P2	P1-5
Level 1	0	0	0	0	0	0
Level 2	100	100	50	100	20	100
Level 3	500	500	60	500	30	500

Table 13. Boundaries of each criterion (mn_i, md_i, mx_i).

Criteria i	mn_i	md_i	mx_i	Units
$z1$ (Cost)	-	87574	94096	$
$z2$ (Quality score)	28891	32798	-	Score
$z3$ (Delivery lateness)	-	12101	13298	Day

Table 14. Capacity (h_{jk}), minimum order quantity (MOQ (o_{jk})) and Min % of demand to be purchased (%Demand r_{jk})).

Data	P/S	S1	S2	S3	S4	S5
Capacity (h_{jk})	P1	1000	500	400	1500	700
	P2	0	50	0	40	0
	P3	300	1000	100	0	0
	P4	0	0	500	2000	600
	P5	0	0	0	3000	2000
MOQ (o_{jk})	P1	0	0	0	0	0
	P2	0	0	0	0	0
	P3	0	10	0	0	0
	P4	0	0	0	0	0
	P5	0	0	0	100	0
%Demand (r_{jk})	P1	0	0	0	0	0
	P2	0	0	0	0	0
	P3	0	0.1	0	0	0
	P4	0	0	0	0	0
	P5	0	0	0	0	0.05

α_{ck} 1 if quantity discount level c is selected for supplier k; 0 otherwise (unitless)
β_{ck} 1 if volume discount level c is selected for supplier k; 0 otherwise (unitless)
λ_i satisfaction level of criteria i ; cost, quality and delivery lateness (unitless)
s overall satisfaction level formulated by weighted maximin model (unitless)
sl the minimum of satisfaction levels of all criteria (unitless)
γ satisfaction level of fuzzy demand from all products (unitless)
z_{jn} 1 if demand level n is selected for product j ; 0 otherwise (unitless)
sld_j satisfaction level of fuzzy demand of each product j (unitless)
d_{jn} total demand of product j at level n (units)

The six models and constraints are illustrated as follows.

3.2.1. Additive Model

In this model, we assume that all criteria are equally important. The model aims to maximize the average satisfaction levels of all criteria including the achievement level of fuzzy demand. The objective function is shown in Eq. (4).

Maximize

$$\frac{\sum_i \lambda_i + \gamma}{4} \quad \text{. . . . . (4)}$$

where λ_i is the satisfaction level of criterion i, i.e., cost, quality, and delivery lateness. γ is the satisfaction level of demand.

Price discount :

In quantity discount constraints, f_k is equal to 1. Eqs. (5) and (6) show that the total purchasing cost (t_{cjk}) corresponds to the purchased quantity (x_{cjkn}) and unit price at a particular discount level (p_{cjk}). In addition, only one quantity level can be selected, as defined by Eq. (7). f_k is equal to 0 when the volume discount policy is used, as shown in Eq. (8). Eq. (9) indicates that the business volume (a_{ck}) corresponds to the purchased quantity (x_{cjkn}) and the unit price $(z1_{jk})$. Note that only one discount level can be selected as defined by Eq. (10).

$$\sum_c t_{cjk} \cdot f_k = \sum_c \sum_n p_{cjk} \cdot x_{cjkn} \cdot f_k \quad \forall j,k \quad \ldots \quad (5)$$

$$e_{c-1,jk} \cdot \alpha_{ck} \cdot f_k \leq \sum_j x_{cjkn} \cdot f_k < e_{cjk} \cdot \alpha_{ck} \cdot f_k \quad \forall c,k,n \quad \ldots \quad (6)$$

$$\sum_c \alpha_{ck} \cdot f_k \leq 1 \quad \forall k \quad \ldots \quad (7)$$

$$\sum_c a_{ck} \cdot (1 - f_k) = \sum_c \sum_j \sum_n z1_{jk} \cdot x_{cjkn} \cdot (1 - f_k) \ \forall k (8)$$

$$b_{c-1,k} \cdot \beta_{ck} \cdot (1 - f_k) \leq a_{ck} \cdot (1 - f_k) < b_{ck} \cdot \beta_{ck} \cdot (1 - f_k) \ \forall c,j,k \quad (9)$$

$$\sum_c \beta_{ck} \cdot (1 - f_k) \leq 1 \quad \forall k \quad \ldots \quad (10)$$

Available supplier: A supplier may supply only some products, but not all of the products.

$$\pi_{jk} \leq sr_{jk} \quad \forall j,k \quad \ldots \quad (11)$$

Capacity: The total purchasing quantity x_{cjkn} must be less than the supply capacity h_{jk} and it is active only if supplier k is selected to supply product j ($\pi_{jk} = 1$).

$$\sum_c \sum_n x_{cjkn} \leq h_{jk} \cdot \pi_{jk} \quad \forall j,k \quad \ldots \quad (12)$$

Limited number of suppliers: The number of suppliers cannot exceed the available suppliers.

$$l_j \leq \sum_k \pi_{jk} < u_j \quad \forall j \quad \ldots \quad (13)$$

Minimum order quantity: The total purchasing quantity x_{cjkn} must be greater than the required minimum order quantity of product j from supplier k

$$o_{jk} \cdot \pi_{jk} \leq \sum_c \sum_n x_{cjkn} \quad \forall j,k \quad \ldots \quad (14)$$

Relationship: The agreement with supplier k that a firm will purchase product j, at least some percentage of the total demand from supplier k.

$$r_{jk} \cdot \sum_n d_{jn} \leq \sum_c \sum_n x_{cjkn} \quad \forall j,k \quad \ldots \quad (15)$$

Fuzzy demand: Total purchasing quantity x_{cjkn} must be in a range of minimum $bo_{m,j}$ and maximum $bo_{m+1,j}$ demand levels, and only one demand level z_{jn} must be selected.

$$bo_{mj} \cdot z_{jn} \leq d_{jn} < bo_{m+1,j} \cdot z_{jn} \quad \forall j,m,n \quad \ldots \quad (16)$$

$$\sum_c \sum_k x_{cjkn} = d_{jn} \quad \forall j,n \quad \ldots \quad (17)$$

$$\sum_n z_{jn} = 1 \quad \forall j \quad \ldots \quad (18)$$

Satisfaction level: Eqs. (19)–(21) describe the satisfaction levels of cost, quality, and delivery lateness criteria. Eqs. (22)–(24) calculate the satisfaction levels of the fuzzy demand.

$$\lambda_1 \leq \left(mx_1 - \sum_c \sum_j \sum_k t_{cjk} \cdot f_k + \sum_c \sum_k a_{ck} \cdot (1 - g_{ck}) \cdot (1 - f_k) \right) / (mx_1 - md_1) \quad \ldots \quad (19)$$

$$\lambda_2 \leq \frac{\sum_c \sum_j \sum_k \sum_n z2_{jk} \cdot x_{cjkn} - mn_2}{md_2 - mn_2} \quad \ldots \quad (20)$$

$$\lambda_3 \leq \frac{mx_3 - \sum_c \sum_j \sum_k \sum_n z3_{jk} \cdot x_{cjkn}}{mx_3 - md_3} \quad \ldots \quad (21)$$

$$sld_j \leq \frac{bo_{3j} - \sum_n d_{jn}}{bo_{3j} - bo_{2j}} \quad \forall j \quad \ldots \quad (22)$$

$$sld_j \leq \frac{\sum_n d_{jn} - bo_{1j}}{bo_{2j} - bo_{1j}} \quad \forall j \quad \ldots \quad (23)$$

$$\gamma \leq sld_j \quad \forall j \quad \ldots \quad (24)$$

Non-negativity conditions and the range of values: Eqs. (25)–(27) have non-negativity conditions, and a range of values.

$$0 \leq \lambda_i < 1 \quad \forall i \quad \ldots \quad (25)$$

$$0 \leq sld_j < 1 \quad \forall j \quad \ldots \quad (26)$$

$$0 \leq \gamma < 1 \quad \ldots \quad (27)$$

3.2.2. Weighted Additive Model

A basic concept of this model is to assign the relative importance of criteria to the additive model, and maximize the average value of all satisfaction levels.

Maximize

$$\left(\sum_i w_i \cdot \lambda_i \right) + (\sigma \cdot \gamma) \quad \ldots \quad (28)$$

All constraints of this model are illustrated in Eqs. (2)–(24).

3.2.3. Maximin Model

Different from the additive model, the maximin model attempts to maximize the minimum satisfaction levels of all criteria. In this model, all criteria are equally important.

Maximize

$$sl \quad \ldots \quad (29)$$

In this model, the constraints are defined by Eqs. (5)–(27). Additionally, three non-negativity conditions are added, as shown in Eqs. (30)–(32).

$$sl \leq \gamma \quad \ldots \quad (30)$$

$$sl \leq \lambda_i \quad \forall i \quad \ldots \quad (31)$$

$$0 \leq sl < 1 \quad \ldots \quad (32)$$

3.2.4. Weighted Maximin Model

This model is adjusted from the maximin model by taking criteria weights into account. It interesting here to notice that the equations of satisfaction levels, Eqs. (19)–(24), are changed to Eqs. (34)–(39).

Maximize

$$s \quad (33)$$

For the model constraints, they are subjected to Eqs. (2)–(15), and Eq. (23), together with the added constraints, as follows.

$$w_1 \cdot s \leq \left(mx_1 - \sum_c \sum_j \sum_k t_{cjk} \cdot f_k + \sum_c \sum_k a_{ck} \cdot (1 - g_{ck}) \cdot (1 - f_k) \right) / (mx_1 - md_1) \quad (34)$$

$$w_2 \cdot s \leq \frac{\sum_c \sum_j \sum_k \sum_n z2_{jk} \cdot x_{cjkn} - mn_2}{md_2 - mn_2} \quad (35)$$

$$w_3 \cdot s \leq \frac{mx_3 - \sum_c \sum_j \sum_k \sum_n z3_{jk} \cdot x_{cjkn}}{mx_3 - md_3} \quad (36)$$

$$\sigma \cdot sld_j \leq \frac{bo_{3j} - \sum_n d_{jn}}{bo_{3j} - bo_{2j}} \quad (37)$$

$$\sigma \cdot sld_j \leq \frac{\sum_n d_{jn} - bo_{1j}}{bo_{2j} - bo_{1j}} \quad \forall j \quad (38)$$

$$s \leq sld_j \quad \forall j \quad (39)$$

$$0 \leq s < 1 \quad (40)$$

3.2.5. Augmented Model

To maximize the average satisfaction levels and the minimum satisfaction levels of all criteria at the same time, the objective function is changed to Eq. (41).

Maximize

$$\frac{sl + \left(\sum_i \lambda_i + \gamma \right)}{4} \quad (41)$$

All constraints are drawn from the maximin model Eqs. (5)–(27) and Eqs. (30)–(32).

3.2.6. Weighted Augmented Model

The weighted augmented model is developed from the augmented model. Therefore, all constraints are the same as the augmented model.

Maximize

$$sl + \left(\sum_i w_i \cdot \lambda_i + \sigma \cdot \gamma \right) \quad (42)$$

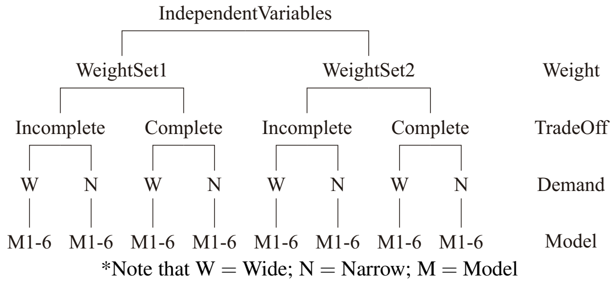

Fig. 2. Experimental factors of each data set.

4. Design of Experiment to Statistically Analyze Effects of Aggregation Operators

To statistically analyze the sensitivities of the optimal solutions and the advantages of aggregation operators, five data sets are generated by varying randomly the capacity, number of suppliers, minimum order quantity, and relationships with suppliers. In designing the experiment, independent and dependent variables are defined. Models investigate how independent variables significantly affect dependent variables. The experimental results are analyzed by MINITAB software.

Independent variables: Four independent variables are considered in this study: (1) two sets of weights as defined in **Table 4**, (2) two types of demand ranges (wide and narrow demand ranges) as defined in **Table 6**, (3) six models as shown in **Fig. 1**, and (4) two types of trade-offs (Incomplete and Complete trade-offs), as shown in **Table 7**. An incomplete trade-off means that there are some dominant suppliers. For example, supplier 1 is considered as a dominant supplier if supplier 1 provides the lowest cost, highest quality, and lowest delivery lateness. Each data set consists of 48 combinations, as illustrated in **Fig. 2**.

Dependent variables: The dependent variables are the performance indicators and are used as responses in MINITAB software. The average satisfaction level and the lowest satisfaction level are two responses in this study.

5. Results and Discussion

Results are evaluated in four aspects, namely, verification of reasonable results, average satisfaction level, lowest satisfaction level, dominated solution, and how to select the aggregation operator to match the risk preferences of decision makers.

5.1. Reasonable Result Verification

From **Table 15**, it can be seen that the model yields reasonable results, as follows. Product 4 (P4) is supplied by 3 suppliers. If there is only a cost criterion, all units must be ordered from S5 due to the lowest price offered. As multiple criteria are concerned, the model is required to make

Table 15. Optimal purchasing quantity of weighted additive technique: weight set1, complete trade-off, narrow demand range.

P/S	S1	S2	S3	S4	S5
P1	-	50	-	-	450
P2	-	-	-	30	-
P3	-	10	90	-	-
P4	-	-	179	471	50
P5	-	-	-	500	2000

```
Grouping Information Using Tukey's
test and 95.0% Confidence

Model  N  Mean  Grouping
1      8  0.63  A
4      8  0.62  A B
6      8  0.60    B
3      8  0.60    B
2      8  0.54      C
5      8  0.50        D

Means that do not share a letter
are significantly different.
```

Fig. 3. Grouping for the average satisfaction level.

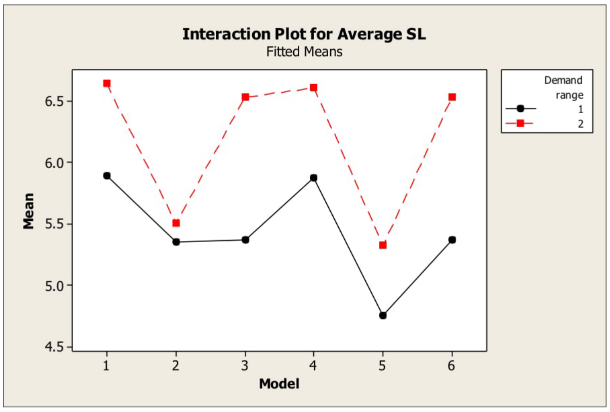

Fig. 4. Interaction plot of model and demand range for the average satisfaction level.

```
Grouping Information Using Tukey's
test and 95.0% Confidence

Model  N  Mean  Grouping
2      8  0.38  A
3      8  0.38  A
6      8  0.38  A
1      8  0.20    B
4      8  0.17      C
5      8  0.12        D

Means that do not share a letter
are significantly different.
```

Fig. 5. Grouping for the lowest satisfaction level.

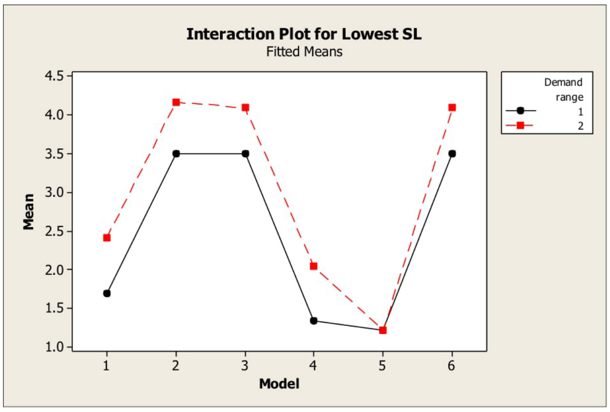

Fig. 6. Interaction plot of model and demand range for the lowest satisfaction level.

trade-offs among criteria with respect to assigned weights from decision makers. As can be seen from Table 4, the quality score of S4 is greater than S5 (10:5) and the delivery lateness of S5 is less than S4 (4:5). Thus, to achieve the highest satisfaction of decision makers, decision makers purchase P4 at a slightly higher price and gain much better quality and a slightly worse delivery lateness. In addition, as the fuzzy demand has the highest weight (32%), decision makers prefer to purchase at an amount close to the predicted demand. Hence, the total demand of P4 in this model is exactly 700 units.

5.2. Level of Average Satisfaction

By means of statistical analysis, a two-level full factorial design of experiment is applied and each insignificant factor is gradually deleted, beginning with the highest p-value of interaction factors, until only significant factors are left. The results show that models with additive operators (Model 1 and 4) have significantly higher average satisfaction level than those with augmented operators (Model 3 and 6) and maximin operators (Model 2 and 5) in both weight and without weight's environments, as presented by Tukey's test in **Fig. 3**. Since the model and demand range have significant interaction effects, an interaction plot is shown in **Fig. 4**. This interaction effect indicates that, for all models, a wider range of demand provides a higher average satisfaction level than a narrower one.

5.3. Level of the Lowest Satisfaction

In **Fig. 5**, the maximin aggregation operator (Model 2) has a significantly higher lowest satisfaction level than models based on additive operators (Model 1 and 4). A benefit of the maximin operator is to avoid very bad performance. Although the weighted maximin model is developed using the maximin operator, it provides the lowest satisfaction level (Lowest satisfaction level = 0.1), instead of the highest satisfaction level (Highest satisfaction level = 0.4). In addition, the results show that an interaction between the method and the demand range is statistically significant. This is because the model has more ability to search for a better solution when the demand range is wider, as presented in **Fig. 6**.

Table 16. Dominated solution (weight set 2, complete trade-off, narrow demand range).

Model/Criteria	Cost	Quality	Delivery lateness	Demand	Dominated solution
Additive	0.99	0.6	0.18	0.57	No
Maximin	0.99	0.46	0.34	0.34	No
Augmented	0.99	0.46	0.34	0.34	No
Weighted additive	1	0.63	0.11	0.6	No
Weighted maximin	0.99	0.33	0.11	0.36	Yes
Weighted augmented	0.99	0.46	0.34	0.34	No

Table 17. Tukey's group, based on average and lowest satisfaction levels.

Model/Indicator	Average satisfaction level (SL)			Lowest satisfaction level (SL)		
	Tukey's group	Average SL	Rank	Tukey's group	lowest SL	Rank
Additive	A	0.63	1	B	0.20	2
Weighted additive	A B	0.62	1,2	C	0.17	3
Maximin	C	0.54	3	A	0.38	1
Weighted maximin	D	0.50	-	D	0.12	-
Augmented	B	0.60	2	A	0.38	1
Weighted augmented	B	0.60	2	A	0.38	1

Table 18. Suitable models for different risk preferences of decision makers.

Model/Risk preference	Risk-taking	Risk-averse	Risk-neutral
Additive	✓	-	✓
Weighted additive	✓	-	-
Maximin	-	✓	-
Weighted maximin	N/A	N/A	N/A
Augmented	-	✓	✓
Weighted augmented	-	✓	✓

5.4. Dominated Solution

A solution is considered a dominated solution whenever the satisfaction levels of all criteria are worse than or the same as those of other solutions. The results show that all models, except the weighted maximin model, do not provide any dominated solutions, or we can say that other models yield the Pareto optimal solutions. As presented in **Table 16**, it is noticed that every satisfaction level of the weighted maximin model is lower than the weighted additive model. This is because of its algorithm. If the satisfaction levels of all criteria are equal to their assigned weights, the weighted maximin model will get the optimal solution (the sum of all satisfaction levels = 1.0). There is no effort to strive for a better solution. Thus, there is a high chance that the weighted maximin model will be dominated by the others since the sum of satisfaction levels of other models can be greater than one.

5.5. How to Select the Aggregation Operator to Match the Risk Preferences of Decision Makers

Since the weighted maximin model is dominated by another model, it is not analyzed in this section. Hence, only five models are analyzed. As mentioned above, according to the characteristics of decision makers based on risk preference, the risk-taking decision makers focus only on a high average satisfaction level. They do not mind if there is a risk to have a zero satisfaction on a certain criterion. From **Table 17**, we notice that both the additive and weighted additive models have the highest average SL and are ranked as the first group. It can be inferred that the additive and weighted additive models are suitable for risk-taking decision makers. In other words, we suggest that the additive aggregation operator is suitable for risk-taking decision makers. In contrast, the risk-averse decision makers concentrate on the lowest satisfaction level. The maximin, augmented, and weighted augmented models are the best group of the lowest SL. Therefore, they are suitable for risk-averse decision makers. Finally, risk-neutral decision makers do not accept a solution with the worst average SL or the worst lowest SL. Hence, the additive, augmented, and weighted augmented models are recommended for them.

6. Concluding Remarks

In this paper, we have proposed realistic FMOLP models with volume and quantity discounts under fuzzy demand and how to select a proper aggregation operator and the model, based on risk preference of decision makers. The effects of the aggregation operator are statistically analyzed. The results reveal that the solutions are reasonable with different sets of input parameters. From **Table 18**, if all criteria are equally important, results show that the additive model is suitable for both risk-taking and risk-neutral decision makers while the maximin model is suggested for risk-averse decision makers. Finally, the augmented model is suitable for risk-averse and risk-neutral decision makers. However, when weights of criteria are different, risk-taking decision makers prefer the weighted additive model while risk-averse and risk-neutral decision makers prefer the weighted augmented model. It is also important here to note that the weighted maximin model should be applied with caution since it may generate a dominated solution.

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