Grain Acceleration in the Interstellar Medium (ISM)

DESY Summer Student Programme, 2017

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September 4, 2017

Abstract

We simulate the grain's acceleration in the turbulent electromagnetic fields. We take into account the gyroresonance of grains with MHD waves in each mode (Alfvenic, slow and fast modes). Gyroresonance can accelerate and scatter the grains. We ignore the scattering process because of low efficiency. We also take into account the hydrodynamic force by collisions of grains with plasma fluid. We find that the fast mode gives the most contribution to grain acceleration. And the strength of magnetic field can determine the relative importance of gyroresonance and hydrodynamical drag.

Keywords: MHD turbulence, Gyroresonance, Hydrodynamical drag

Contents

1 Introduction

Dust grains are considered an important component of the Interstellar Medium (ISM). Even though they are only 1% of mass of the ISM, they can catalyze the formation of gas-phase molecules such as H_2 which involve the collapse and formation of stars and planets. Grain acceleration also explain overabundance of refractory elements in galactic cosmic rays. The stochastic acceleration can be acted as preacceleration mechanism of the ions released from the grains which can be further accelerated in the shock (See YLD04). In addition, dusts play an important role in the ISM by providing insight to star formation activity. Therefore, many studies of acceleration of the grains in the astrophysical plasma become important. We believe that the various processes such as radiation, ambipolar diffusion and gravitational sedimentation can not provide enough random velocity to affect the grain's interstellar population. In this work, the acceleration of the grains comes from the interaction with magnetohydrodynamic (MHD) waves and hydrodynamical drag.

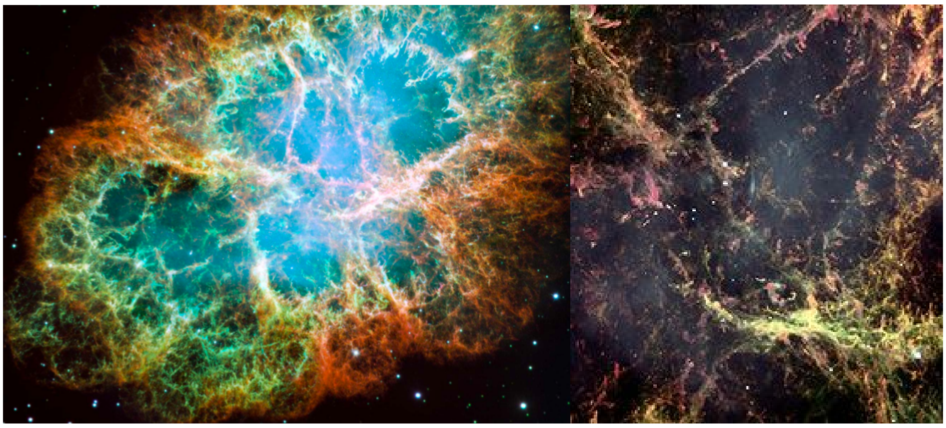

Figure 1: Crab Nebula, an example of interstellar medium. It is a supernova remnant with multi-scale structure. Credit Image: wiki/Crabnebula

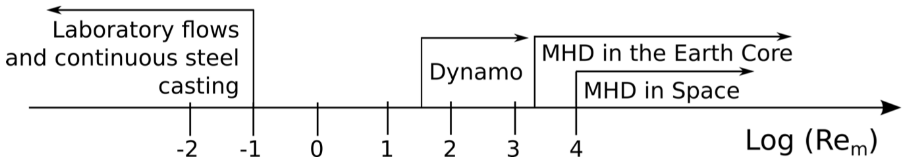

Figure 2: Magnetic Reynold number Re_m in log scale for different processes. $Re_m \geq 10^3$ for astrophysical objects

ISM is magnetized and turbulent. Charged grains can be scattered and accelerated by having interactions with magnetized and turbulent medium. Astrophysical plasma has very large magnetic Reynold numbers ($R_m = \mu_0 \sigma v L$) due to large-scale (L) involved (Figure 2). We can then assume that the magnetic field convects with the plasma fluid. This is referred to be "flux freezing". The movement of grains inside ISM then affect the magnetic configuration. In plasma, electrons and ions are locked together by electrostatic forces as the drift velocities are small. Thus, we can define the plasma as made of a single component.

2 Theoretical Background

2.1 Magnetohydrodynamics

In magnetohydrodynamics (MHD), we make an approximation that the electrons and ions are locked together. Thus, in the frame of fluid, the charge density (ρ') is much less than current density ($\mathbf{j'/c}$). This makes the electric field less than magnetic field ($\mathbf{E} << \mathbf{B}$). Another assumption is that the fluid velocity is nonrelativistic. In such a case, we can rewrite the Maxwell's equations and equation of continuity as MHD equations as follows:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \tag{3}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \frac{1}{4\pi\rho}(\boldsymbol{\nabla} \times B) \times B \tag{4}$$

where $B, \rho, \mathbf{v}$ are magnetic field, fluid density and flow velocity.

We consider the plasma fluid to have homogeneous magnetic field, density and pressure with fluid initial velocity at rest. We can split these physical quantities into 2 parts which are mean values and fluctuation terms.

$$\mathbf{B} = \mathbf{B} + \mathbf{h}, \qquad \rho = \rho + \rho', \qquad \mathbf{p} = \mathbf{p} + \mathbf{p'} \tag{5}$$

From these small perturbation, together with wave solution form to solve MHD equations, we get 3 different modes of oscillations.

Alfven Waves

The Alfven wave is described by dispersion relation

$$\omega^2 = v_A^2 k^2 cos^2\alpha \tag{6}$$

where α is the angle between magnetic field ($\mathbf{B}$) and wave propagation direction ($\mathbf{k}$) and $v_A = (B^2/4\pi\rho)^{1/2}$ is called the Alfven speed. This wave corresponds to the magnetic tension. Alfven wave is mostly incompressive unlike other modes.

Fast and Slow waves

The dispersion relation becomes

$$\omega^2 = \frac{k^2}{2}\left((v_A^2 + u_0^2) \pm \left[(v_A^2 + u_0^2)^2 - 4v_A^2 u_0^2 \cos^2\alpha \right] \right) \tag{7}$$

where u_0 is sound speed. The plus sign represent the fast MHD wave with higher phase velocity and the minus sign shows the slow MHD wave with lower phase velocity. The phase variation in pressure and magnetic field oscillation are correlated for fast mode and anti-correlated for slow mode (Figure 4.) so that the restoring force of slow mode is weaker than that of fast mode.

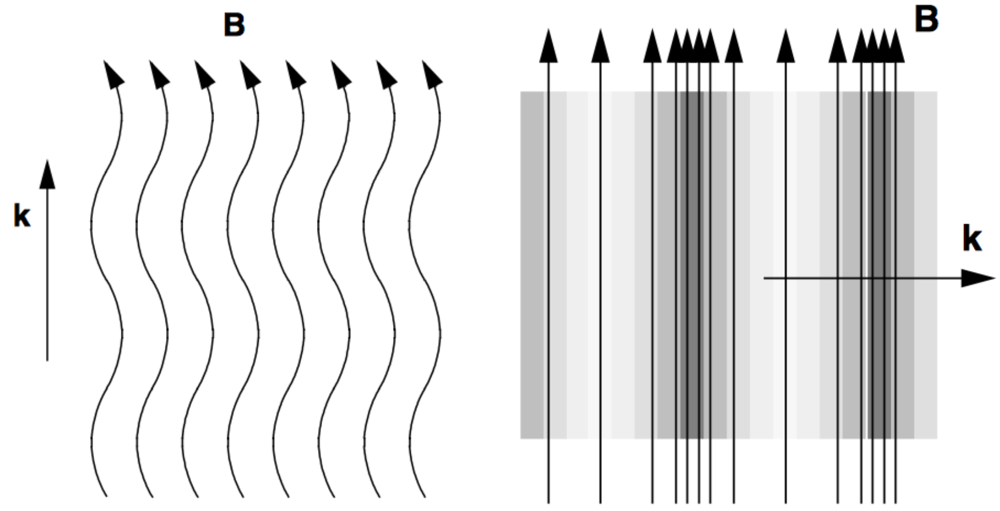

Figure 3: Left: the propagation of Alfven wave. The fluctuation is perpendicular to the field line d ; Right: the propagation of magnetoacoustic wave (Fast and Slow wave). We can see the compression of the medium.

2.2 Acceleration of the grains

When the grain is moving inside the plasma, there are 2 main forces exerting on the grains. There are electromagnetic forces from gyroresonance and hydrodrag force which is due to collisions with the neutrals and ions in plasma

The grain's acceleration occurs from the gyroresonance interaction with MHD waves. The resonance condition is

$$\omega - k_{\|} v\mu = n\Omega \tag{8}$$

where ω is the wave frequency,$k_{\|}$ is the parallel component of wavevector along the magnetic field,v is the particle velocity,μ is the cosine of pitch angle.

According to the interaction of grains with MHD waves, there are scattering and acceleration processes. In this work, we neglect the scattering process as it is less efficient for sub-Alfvenic grains with non-parallel motion to the magnetic field.

I. Electromagnetic Force

In MHD case, we can neglect the electric field in the lab frame. The Lorentz force on a charged grains is given by

$$f_L = \frac{1}{4\pi}(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} \tag{9}$$

$$= \frac{1}{4\pi}(\mathbf{B} \cdot \nabla)\mathbf{B} - \frac{\mathbf{1}}{\mathbf{8}\pi}\nabla^{\mathbf{2}}\mathbf{B} \tag{10}$$

where f_L is the Lorentz force per unit volume

The first term on the right hand side corresponds to the magnetic tension in the direction of magnetic field. The second term can be thought of as magnetic pressure.

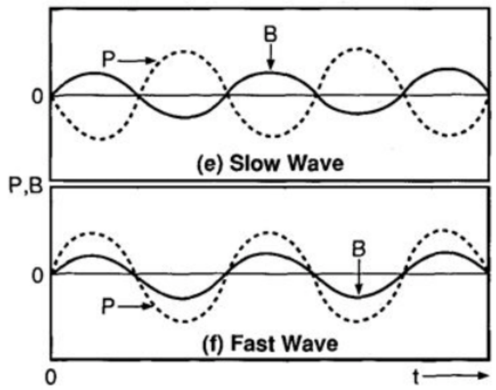

Figure 4: Shows the phase of pressure fluctuation and magnetic field. In slow wave (up), the fluctuation is out of phase with magnetic field. In fast wave (down) the fluctuation is in phase with magnetic field.

II. Hydrodynamical Drag Force

When the grains move into the fluid. They will be exerted by drag force due to the viscosity of the fluid. The hydrodrag force can be written as follows:

$$F_d = -\frac{mv}{t_{\rm drag}} \tag{11}$$

where $t_{\rm drag}$ is the drag time due to collision with atoms in plasma. It is generally defined as $t^0_{\rm drag} = (a\rho_{\rm gr}/n_n)(\pi/8m_n k_B T)^{1/2}$. When the grain's velocity become supersonic, the drag time become $t^s_{drag} = t_{\rm drag}/(0.75 + 0.75c_s^2/v^2 - c_s^3/2v^3 + c_s^4/5v^4)$ where c_s is the sound speed in the plasma.

As from these two forces, the electromagnetic force accelerates grains while hydrodrag force decelerates grains. After certain time, the velocity of grains should converge to constant value.

2.3 Magnetohydrodynamic Turbulence

For ISM, many astrophysical phenomena can drive the turbulence such as supernovae explosions, collimated outflows, plasma instabilities etc. As the astrophysical plasma is turbulent, there will be an energy cascade of the MHD waves. We make an assumption that MHD turbulence follows Kolmogorov's law.

According to the above figure, the turbulence occur from the injection scale and cascade down to cutoff scale. The injection scale is the scale where energy is injected and large eddies are created. These large eddies transfer energy to smaller eddies. This process occur as Kolmogorov's law until the scale in which cascading rate equals to the damping rate. This scale is called cutoff scale. In CNM, the damping process is due to neutral-ion collision. For grains to be accelerated, the scale in which resonance of grain-wave interaction occurs must be larger than cutoff scale.

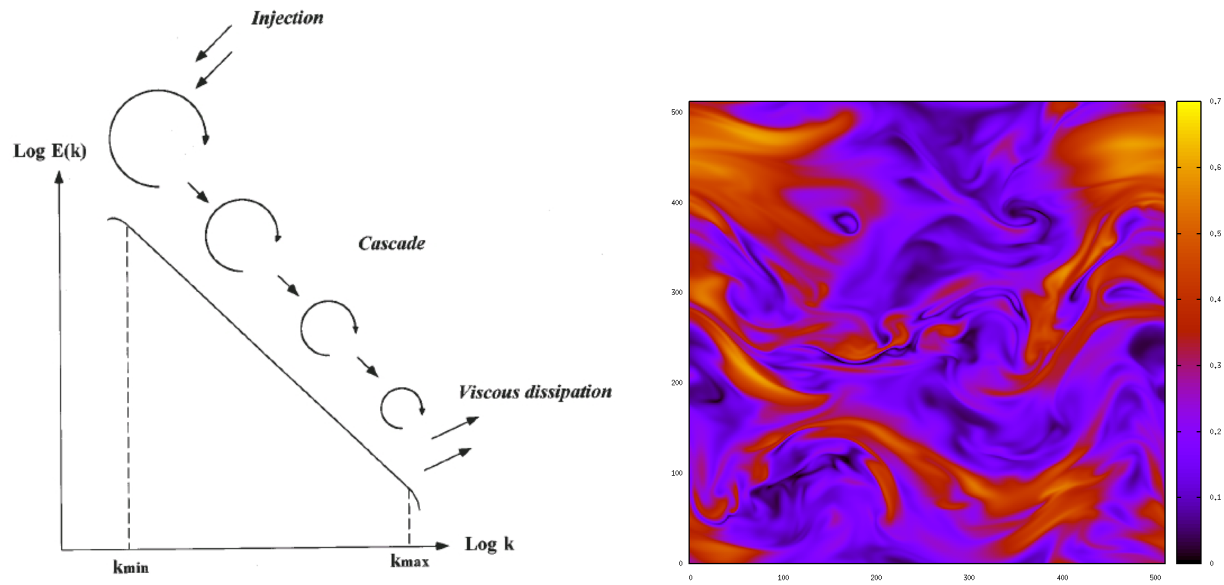

Figure 5: Left: Kolmogorov's theory represents the energy cascade from injection scale to cutoff scale with power law ($E \sim k^{-5/3}$) ; Right: This figure shows example of slice of our magnetic field datacube used in the simulation. We can see many scales of eddies there. The colorbar represents the strength of magnetic field.

2.4 Grain Charging

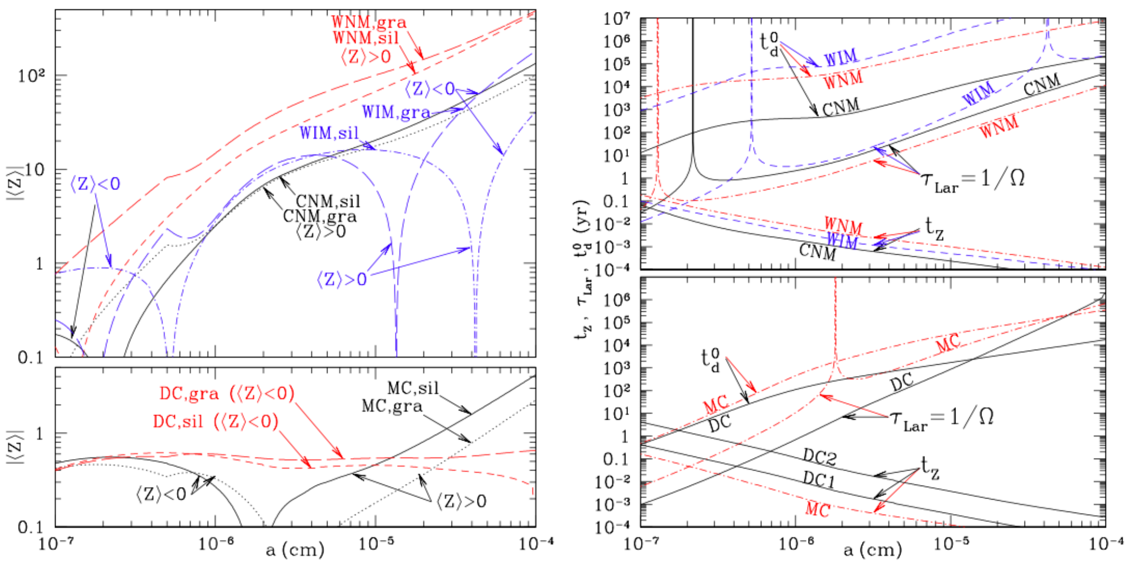

Figure 6: Left: The figure shows the average electric charge number in each phase of ISM.; Right: The figure shows the charge fluctuation time (t_z) versus other dynamical times.

As the grains are subjected to the Lorentz force, we should look at the charge of the grains. The net charge of the grains depend on the rate of collision of electrons which add negative charge and with ions with ions which remove negative charge. Moreover, the photoelectric emission also removes the negative charge from the grains. As a consequence, grains have a fluctuation time (t_z).

According to the above figure, we apply the radius of the grain greater than 10^{-5} cm in our simulation. Because the fluctuation time is much shorter than other dynamical times so that we can neglect the charge fluctuation and assume that electric charge number $Z =< Z >$ which is obtained from Figure 6.

3 Simulation Method

After getting the physical values for each parameter in CNM from YLD04, we can apply the initial values in the simulation. We write the program in fortran language and plot the results by using Gnuplot.

ISM	CNM
T(K)	100
$n_{\rm H}({\rm cm}^{-3})$	30
$n_e({\rm cm}^{-3})$	0.03
G_{UV}	1
B(μG)	6
L(pc)	0.64*
$V = V_A$(km/s)	2*
damping	neutral-ion
$k_c({\rm cm}^{-1})$	7×10^{-15}

Figure 7: This figure shows the physical parameters of the ISM in the CNM phase from YLD04. The calculated sound speed $c_s = 2.1$ km/s

The code does the following steps:

1. Obtain the magnetic field and velocity field datacube for MHD turbulence of different modes (original, fast, Alfven, slow) from PENCIL

2. Interpolate magnetic field and velocity field from each point.

3. Inject 200 grains with radius (a $= 3 \times 10^5$ cm) into the datacube with random initial positions and initial pitch angle. The initial velocities are 10^5 cm/s.

4. Update the grains's acceleration from electromagnetic force and hydrodynamical drag force in each time.

5. Save the grains's positions and velocities at each time

4 Results

After running the simulation, we plot the trajectory of the grains in 3-dimension

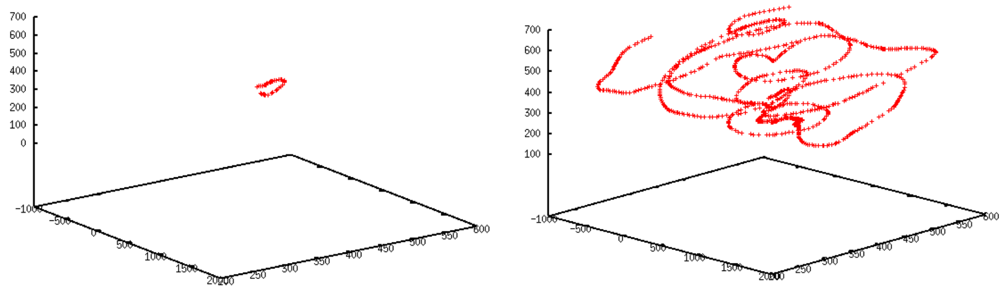

Figure 8: Left: the trajectory of a grain from the beginning to t = 50 ; Right: the trajectory of a grain from the beginning to t = 1000.

As the grains are subjected to magnetic force in the plasma, grains start to gyrate. We can see that the Larmor radius gets larger at later time. As the Larmor radius is proportional to the particle's velocity ($R_L = \frac{mv}{qB}$), the velocity then increases with time. We can see the acceleration of the grains. This acceleration is from the fluctuation of magnetic field the plasma which generates the electric field to accelerate the particle. Moreover, there is also the hydrodrag force which decelerates the grains. From the equation (11), we can see that the hydrodrag force gets higher with the grain's velocity. Finally, the grain's velocity will converge to constant value.

Next, we simulate with number of particles = 100 to observe the ensemble properties in 3 different modes and in total mode datacube.

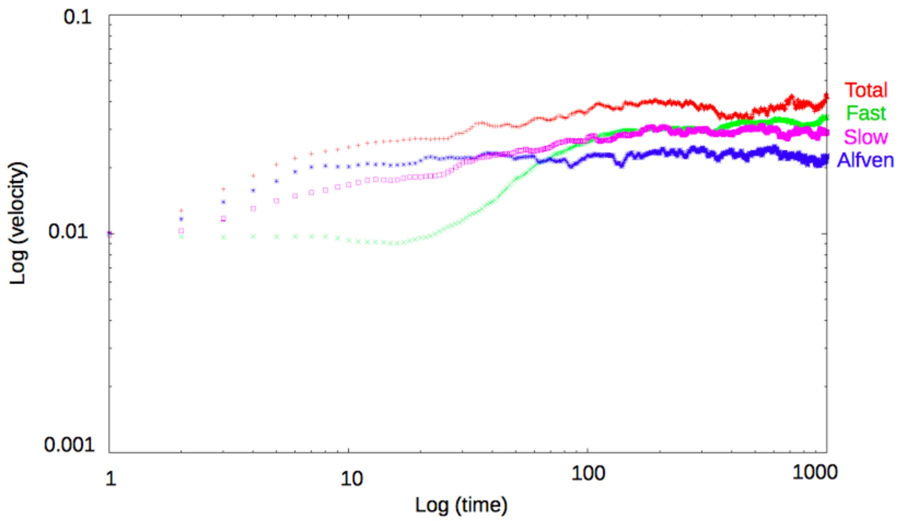

Figure 9: This figure shows the average velocity of the grain with $N = 200$ versus time in log scale. The red line represents the grain's velocity in MHD datacube. The green line shows the the grain's velocity in fast MHD datacube. The purple line shows the the grain's velocity in slow MHD datacube. The blue line shows the the grain's velocity in Alfven MHD datacube.

The Figure 9 shows the grain's average velocity (N=200) versus time in total, fast, slow, Alfven modes with $B = 6\mu G$. We can see the combination of 3 modes results in highest final velocity. The grain's final velocity in fast mode is highest. The velocity increase in Alfven mode is small compared to others. If we also plot ($v \propto t^{1/2}$), we can see that increase in slow mode is parallel to this line. This means grains in slow mode follow diffusion process in momentum space.

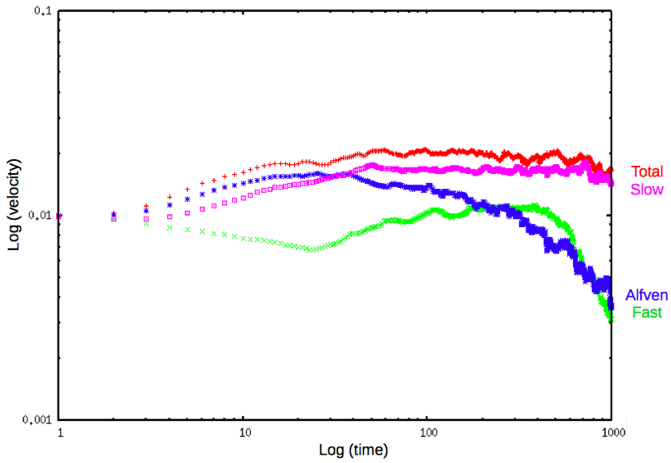

Figure 10: This figure shows the average velocity of the grains with $N = 200$ versus time in log scale with 3 times less than normal magnetic field strength. The red line represents the grain's velocity in MHD datacube. The green line shows the grain's velocity in fast MHD datacube. The purple line shows the the grain's velocity in slow MHD datacube. The blue line shows the the grain's velocity in Alfven MHD datacube.

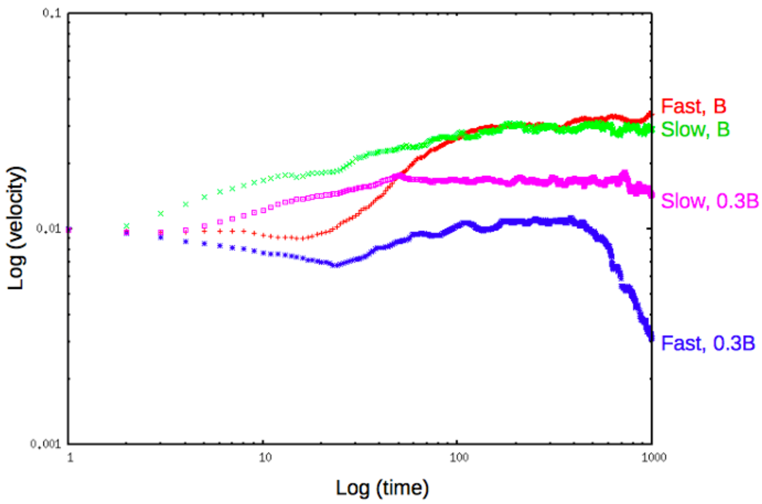

Figure 11: This figure compares the fast mode and slow mode in different magnetic field t. The red line and blue line represent the average grain velocity in fast mode with $B = 6\mu G$ and $B = 2\mu G$ respectively.The green line and purple line represent the average grain velocity in slow mode with $B = 6\mu G$ and $B = 2\mu G$ respectively.

The Figure 10 shows the grain's velocity versus time in total, fast, slow, Alfven modes with $B = 2\mu G$. The grain velocity for fast mode and Alfven drop at later time. They can be interpreted that in the fast mode and Alven mode, the energy of turbulence wave are small at weak magnetic field. The hydrodrag term dominates so as the grain's velocity decreases. In Figure 12, we use the power spectrum of each turbulent mode from the datacube. We can see all lines follow the same energy cascade. It can be seen that the Fast mode has the lowest energy at injection scale.

If we compare the grain's velocity in fast mode and slow mode with different magnetic field strength (Figure 11), we can see the higher final velocity in stronger magnetic field strength in both fast and slow modes. This is because the hydrodrag decreases with the strength of magnetic field. In magnetically dominant regions, gyroresonance is dominant. In weakly magnetized regions, the hydrodrag dominates.

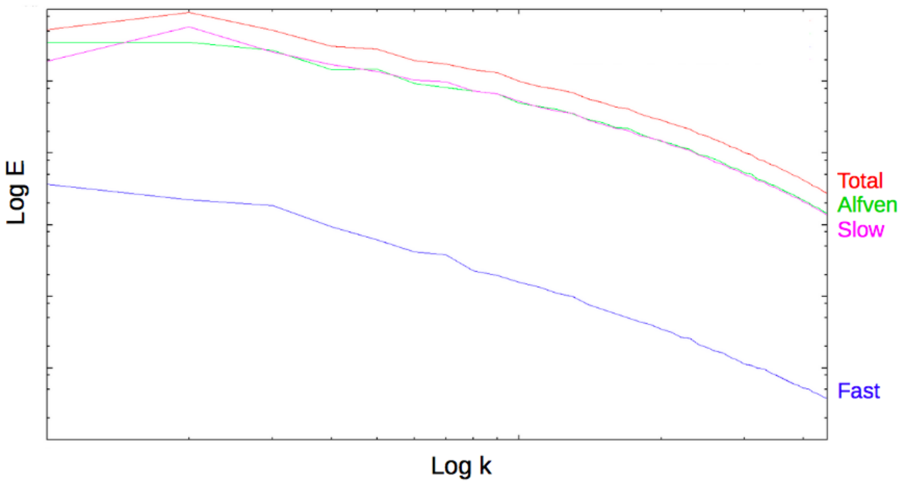

Figure 12: This figure shows the power spectrum of one of our datacube, the original (red), Alfven mode (green),Slow mode (purple) and Fast mode (blue)

In CNM, we can see that grains travel with highest speed in fast mode. The reason is that fast mode turbulent is isotropic. The Alfven mode and slow mode are not important because of their anisotropy. The eddies are elongated along the magnetic field. This makes $k_{\parallel} < k_{\perp}$ which means the there is less energy at the resonance point. This is also consistent with the result in YL03.

5 Conclusions

In this work, we simulate the grain's motion in the Cold Neutral Medium (CNM). The gyroresonance interaction leads to scattering and acceleration of the grains. For sub-Alfvenic grains, we can neglect the scattering process because of low efficiency (YL03). The fast mode is the dominant contribution to grains' acceleration while Alfven mode is not important. The anisotropy of Alfven mode and Slow mode leads to the suppression of gyroresonance.

The strength of magnetic field determines the relative importance of gyroresonance and hydrodynamical drag. In highly magnetized medium, the gyroresonance dominates. In small magnetized medium, the hydrodynamic drag dominates as it decreases with magnetic field strength. The condition for gyroresonance ($k_{res} < k_c$) determines the critical size of the grains. For smaller grains, the gyroresonance gets damped at resonance frequency so that the grains cannot be accelerated.

In future work, we can try with evolving magnetic field and velocity field datacube. Because the grains are non-relativistic particles, we may take into account the evolution of magnetic field as well.

Acknowledgements

I would like to thank my supervisor Heshou Zhang who taught and supported me during this project . Specially thanks to Prof. Huirong Yan and Reinaldo dos Santos de Lima for guiding me in this project. Furthermore, I would like to thank National Science and Technology Development Academy, Thailand for giving me the opportunity to participate in this program.

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